Another equation for distance ☺

So far, we have three equations, which can be written as

① $d = \bar{v}t$ ② $\bar{v} = \frac{v_i + v_f}{2}$ ③ $v_f = at + v_i$

By combining these together, we can derive a new equation.

we start with the first equation: $d = \bar{v}\,t$

Then, substitute in the second equation $\left(\bar{v} = \frac{v_i + v_f}{2}\right)$: $d = \left(\frac{v_i + v_f}{2}\right)t$

Next, substitute in the third equation $(v_f = at + v_i)$: $d = \left[\frac{v_i + at + v_i}{2}\right]t$

Finally, we just do some algebra to clean it up:

$$d = \left[\frac{v_i + at + v_i}{2}\right]t$$

$$d = \left[\frac{at + 2v_i}{2}\right]t$$

$$d = \left[\frac{at}{2}\right]t + \left[\frac{2v_i}{2}\right]t$$

$$d = \frac{1}{2}at^2 + v_i t$$

That equation gives us how far something moved d after a time t when it has a constant acceleration a and an initial velocity V_i .

For example: A car, starting from rest, has a constant acceleration of 3 m/s^2. How far does it travel in 4 seconds?

Answer: $V_i = 0 \quad a = 3 \quad t = 4$

so $d = \frac{1}{2}at^2 + V_i t$

$$d = \frac{1}{2}(3)(4)^2 + (0)(4)$$

$$\boxed{d = 24 \text{ m}}$$

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We could have done it the old way, by first finding the final velocity, then the average velocity and then finding the distance:

$V_i = 0 \quad a = 3 \quad t = 4$

1 $V_f = at + V_i \longrightarrow V_f = (3)(4) + 0$

$V_f = 12 \text{ m/s}$

2 $\bar{V} = \frac{V_i + V_f}{2} \longrightarrow \bar{V} = \frac{0 + 12}{2}$

$\bar{V} = 6 \text{ m/s}$

3 $d = \bar{v} t \longrightarrow d = (6)(4)$

$d = 24 \text{ m}$

The new equation is a lot easier.